Project report: Managing solid waste through discrete location analysis: A case study in central Portugal

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Executive summary

Due to the rapid economical growth in Portugal in the 1980's, a national waste management company decided to implement a network for managing solid waste. The network was not sustainable for the long-term as it would be overloaded after a few years. The installation of an incinerator was selected for solving this issue. Antunes et al. [1] proposed a model in 2001 for one of the municipal solid waste (MSW) management system regions to aid in the decision for building the facilities using discrete location analysis. This study finds the most suitable municipalities for building additional transfer stations and the incinerator considering transportation costs.

The model is faithfully reproduced in this study and, using this solution as a starting point, a modification is proposed which leads to a reduction of 5% in the operational costs. This result is for the original data from 2001 [1].

An extension to the original model is proposed where waste generation is updated to a more recent year (2019), transportation costs are also updated and the recyclable materials are now taken into account. There are now more state-of-the-art methods that do not necessary involve burning waste, and as such, the model requires an update to keep up with the most recent waste regulations in the European Union [4] which aim to protect the environment.

The model extension yields the location of recycling centres, transfer stations and the incinerator obtaining a different network compared to the original model which can be explained by more awareness for separating waste in society.

1 Background

The Litoral Centro region is an autonomous MSW subsystem that is comprised of 36 municipalities, occupying an area of 6 700 km^2. The implementation of the system started in 1997 when a company was selected to manage the waste collection, treatment and disposal for this region.

Sanitary landfills and open-air dumps were used for disposing waste in the 1990's. Transfer stations are facilities for collecting and compacting waste. The compacting technology in place allows a 65% reduction in unit transportation costs inside the network [1].

The landfills and open-air dumps were overloading and the possibility to set up additional landfills was ruled out due to the publication of an European Union directive [3] that imposes a severe limit in the quantities of waste to be disposed in these facilities.

Due to this constraint, the waste management company decided to install an incinerator for waste disposal in the Litoral Centro region. This decision was taken without studying nor detailing the location for this facility. This company contracted the University of Coimbra and the University of Aveiro for identifying the best possible location for the incinerator and for possible new transfer stations.

The approach for the study consisted of three sequential stages:

1. Identification of possible new transfer stations location and the municipality where the inicinerator should be built.
2. Inside the municipality, the best community is chosen to locate the incinerator taking into account the cost and the population resistance.
3. Within the best community, select the best industrial site to build the incinerator through a decision making approach.

2 Original model

In this study, the first stage has been reproduced entirely obtaining the same results reported by Antunes et al. [1]. The full model can be found in appendix A. The assumptions used by the authors are the following:

1. All the generated waste that cannot be recycled at the source is eliminated through the incinerator. The waste must be first compacted in a transfer station and hauled to the incinerator, or sent to this latter facility directly without the compacting process.

2. The road distance for the transportation of compacted waste must not exceed a threshold of 125 km. The same constraint is applied for the uncompacted waste with a value of 25 km. The roads considered are different since the vehicles for the collection of waste and for the transfer vary.

3. The number of new transfer station must be as small as possible. Furthermore, they must be of the same type of the already existing ones.

4. Municipalities connected to an existing transfer station will be remain connected to the same one, regardless of the construction of other transfer stations.

5. Only one incinerator is to be built in the whole region.

2.1 Reproduction results

Data was provided by Dr Antunes and Dr Teixeira for the distances between municipalities considering main roads and highways where the waste trucks can drive through and the waste generation for each municipality in 2001. Additionally, shapefiles for the Litoral Centro region were shared by the paper authors.

The model is run using program developed for this project. This program uses a conjunction of Pyomo [8], a Python software package for formulating optimization models, FICO Xpress 8.7 (Xpress Optimizer 34.01.05) [5] as the optimization solver, and other open-source libraries for data handling and visualization purposes. The optimization results are comfortably obtained in approximately 1.9 seconds in a computer with an Intel core i7 8th generation processor and 16 GB of RAM. The full program implementation is detailed in appendix D.

The below table presents the results obtained in this study, which are also the same as reported by Antunes et al. [1]

Table 1: Paper results

Incinerator	**New transfer stations**	**Annual transportation cost (€x10^3)**
Agueda	Ilhavo Coimbra Montemor-o-Velho	1327.4

2.2 Model exploration

While obtaining the results reported in table 1, it was spotted the objective function proposed considers a penalty term m that restricts the model to build more transfer stations. While the objective function has cost terms for transporting compacted and uncompacted waste, the term $m \sum y_k$ does not represent a real cost term, as the m parameter can be tuned to obtain a different weight between the total number of transfer stations and the operational transport costs. The rigurous definition of the m parameter is a large fixed cost equal to the transportation costs that would have been incurred if no new transfer stations were built multiplied by 100 [1]. Dr Antunes and Dr Teixeira were contacted enquiring for this parameter and confirmed the value to be $m = 10^6$.

Considering that the m term is a virtual cost, an alternative approach was explored. Instead of using this penalty term, the objective function is modified as follows:

$$min\ C = \sum_{j \in J} \sum_{k \in K} c_u d_{j,k} q_j w_{j,k} + \sum_{j \in J} \sum_{l \in L} c_u d_{j,l} q_j v_{j,l} + \sum_{k \in K} \sum_{l \in L} c_c d'_{k,l} x_{k,l}$$

To limit the number of stations, a new constraint is added:

$$\sum_{k \in K} y_k \leq N_{TS} \qquad N_{TS} \geq 6$$

Where N_{TS} represents an upper bound for the total number of transfer stations. This parameter has a lower bound of 6, as this is the number of existing transfer stations in the system. The full alternative model can be found in appendix B.

Since this model lacks fixed costs for building new transfer stations, changing the total number of stations N_{TS} to something different than 9 would not be a fair scenario to compare to the original study. Hence, by setting N_{TS} to 9, the same result was obtained as the original study.

The advantage of this approach is that the objective function solely minimizes real transportation costs. A cost analysis can be performed by exploring or updating values to reflect a change of operations in the company.

2.2.1 Alternative model results

After doing an inspection on the distance matrix that links the municipalities to the transfer stations, the threshold was relaxed from 25 to 30 km. This will give more flexibility to the model to assign the municipalities to transfer stations with a slight more freedom, yet without affecting the collection schedules in a significant manner.

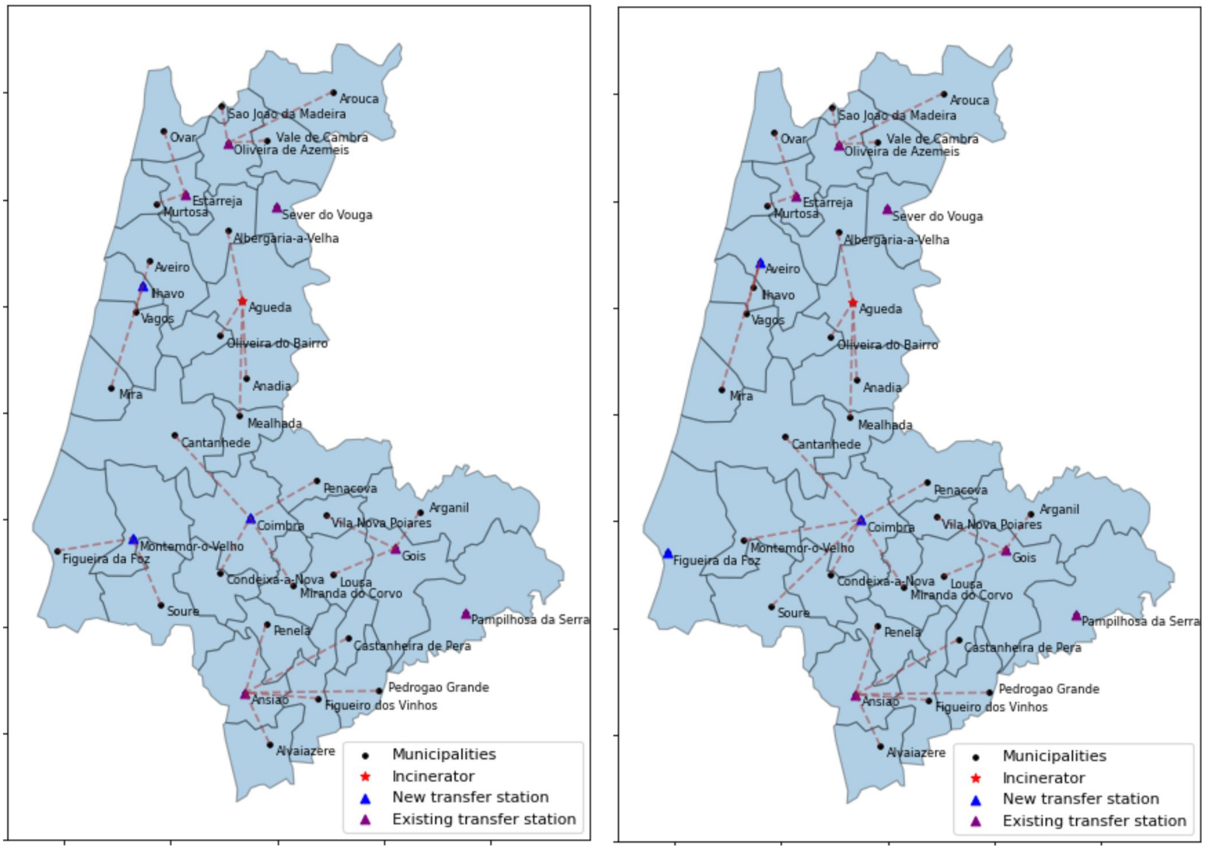
The modified model runs in approximately 2.6 seconds. The results from the modified model with and without the relaxed threshold are presented in the below table:

Table 2: Alternative model results

Max distance (km)	**Incinerator**	**New transfer stations**	**Annual transportation cost (€x10^3)**
30	Agueda	Aveiro Coimbra Figueira da Foz	1260.2
25	Agueda	Ilhavo Coimbra Montemor-o-Velho	1327.4

As it can be seen in table 3, there is a reduction of 5% in operational transportation costs respect to the original results when the maximum distance between municipalities and transfer stations is relaxed to 30 km. This represents a significative improvement for a long-term plan that could be discussed with stakeholders in the waste management company. It is important to remark there are 2 locations not contemplated in the original paper, as this relaxation allows a reconfiguration in the system.

In the below figure, a visualization aid is presented with the results of the original model and the proposed modification using a geospatial representation with the shapefiles shared by Dr Antunes. The map is generated using the `Geopandas` [14] package from Python.



(a) Original results (b) Proposed model results

Figure 1: Visualization comparison between model solutions using Geopandas [14]

3 Model extension

The current trend for solid waste management is to recycle more and use less incineration. Portugal has implemented a circular economy program where waste management companies earn a benefit from organizations such as Ponto Verde [17]. Additionally, the local government subsidizes the collection and separation to encourage recycling.The European Union directive on packaging waste from 2018 [4] states that at least 65% of recyclable materials should be processed as such by 2025. This is the main driving force for extending the original model to include a recycling component in the network.

The most recent data for solid waste and recyclable materials across the 36 municipalities in the Litoral Centro region was obtained for 2019 from the Instituto Nacional de Estatistica from Portugal [11, 12, 13, 16].

Transportation costs are updated from 2001 to 2019 using the Portuguese cost index which considers the inflation rates and economy growth in these years [10]

The full extended model can be found in appendix C. The proposed model considers the additional assumptions:

1. The maximum distance between municipalities and transfer stations, municipalities and recycling centres and municipalities and incinerator is 30 km.

2. The recycling centres are located the farthest away from each other.

3. Transfer stations are located the farthest away from each other.

4. Capacity of recycling centres are the same as transfer stations.

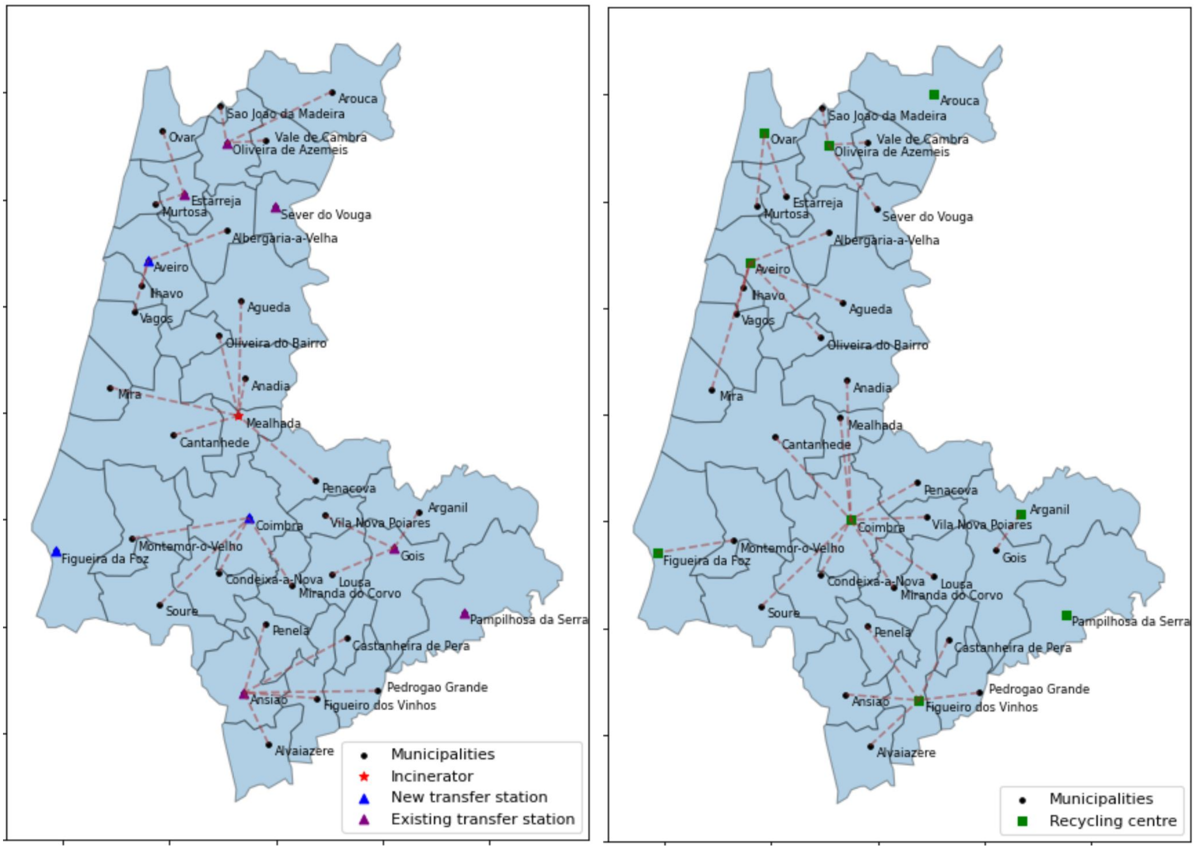
5. The cost of transporting recyclable materials is estimated to be the same as the transport of uncompacted waste.

3.1 Results

The results from the extended model are obtained in approximately 4 minutes.

Table 3: Extended model results

Incinerator	New transfer stations	Recycling centres	Annual transportation cost (€x10^3)
Mealhada	Aveiro Coimbra Figueira da Foz	Arouca Aveiro Oliveira de Azeimeis Ovar Arganil Coimbra Figueira da Foz Pampilhosa da Serra Figueiro dos Vinhos	1693.5



(a) Transfer station and incinerator network (b) Recycling centres network

Figure 2: Network visualization using Geopandas [14]

3.2 Alternative solutions

Other solutions were found for this model; however, these solutions were more expensive or did not offer any additional benefit to the chosen solution. The following solutions were explored:

- Sensitivity analysis for maximum distances for hauling uncompacted waste. The results can be found in appendix C.1
- Only one facility in a municipality case. The details and results can be found in appendix C.2

4 Conclusions

Solid waste management is problem that is being faced by many rapid-growing economies in the world. This project proposes a solution to the challenges experienced in Portugal for locating obnoxious but necessary facilities for proper waste disposal.

The proposed extended model considers the inclusion of recycling facilities in the network for the Litoral Centro region of Portugal. Using a dispersion term in the model, it is possible to obtain a solution that enforces distance spread between facilities.

Municipal solid waste management remains to be a challenge in society. Solid waste management is an essential service in any urban and non-urban society, and yet, there is still resistance from different sources such as government burocracy or society opposition to open more facilities of this kind. Operational Research techniques can be used for formulating and solving this facility location problem and make these facilities *less obnoxious* while giving this problematic an effective solution.

4.1 Future studies

This project could be further extended by considering:

- Weight W parameter in function of the municipality population, aiming to locate the facilities where it is less populated.
- Inclusion of capital costs to open a facility in the objective function. This approach would require consulting with a cost engineering expert to aid in the calculation of the capital costs.

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Appendices

Acknowledgements

We would like to thank Dr António Antunes and Dr João Teixeira from the University of Coimbra for their invaluable support and help. We hope to visit Coimbra when the situation improves worldwide, and take a picture like the one used in our background. Additionally, we would like to thank the open source community that has developed the array of tools used in this project. From typesetting to formulating optimization models, to infinity and beyond.

A Original model

In this section, the first stage of the model from Antunes et al. [1] is presented. The assumptions used by the authors are the following:

1. All the generated waste that cannot be recycled at the source is eliminated through the incinerator. The waste must be first compacted in a transfer station and hauled to the incinerator, or sent to this latter facility directly without the compacting process.

2. The road distance for the transportation of compacted waste must not exceed a threshold of 125 km. The same constraint is applied for the uncompacted waste with a value of 25 km. The roads considered are different since the vehicles for the collection of waste and for the transfer vary.

3. The number of new transfer station must be as small as possible. Furthermore, they must be of the same type of the already existing ones.

4. Municipalities connected to an existing transfer station will be remain connected to the same one, regardless of the construction of other transfer stations.

5. Only one incinerator is to be built in the whole region.

The cost to minimize in this stage is formulated as follows:

$$min\ C = \sum_{j\in J}\sum_{k\in K} c_u d_{j,k} q_j w_{j,k} + \sum_{j\in J}\sum_{l\in L} c_u d_{j,l} q_j v_{j,l} + \sum_{k\in K}\sum_{l\in L} c_c d'_{k,l} x_{k,l} + m\sum_{k\in K} y_k \qquad (1)$$

Constraints

$$\sum_{k\in K} w_{jk} + \sum_{l\in L} v_{jl} = 1 \qquad \forall j \in J$$

$$\sum_{j\in J} q_j w_{jk} = \sum_{l\in L} x_{kl} \qquad \forall k \in K$$

$$w_{jk} \le f_{jk} y_k \qquad \forall j \in J, k \in K$$

$$v_{jl} \le f_{jl} z_l \qquad \forall j \in J, l \in L$$

$$x_{kl} \le g_{kl} q z_l \quad \forall k \in K, l \in L$$

$$\sum_{j\in J} q_j w_{jk} \le s_k y_k \quad \forall k \in K$$

$$\sum_{l\in L} z_l = 1$$

$$w_{jk} = w^0_{jk} \quad \forall j \in J^1, k \in K^1$$

$$w_{jk}, v_{jl}, y_k, z_l \in \{0,1\}, x_{kl} \ge 0 \qquad \forall j \in J, k \in K, l \in L$$

Decision variables

$v_{j,l}$: binary variable that indicates if the municipality j hauls uncompacted waste to the incinerator located in l

$w_{j,k}$: binary variable that indicates if the municipality j hauls uncompacted waste to a transfer station located in k

$x_{k,l}$: quantity of waste processed from the connected municipalities at the transfer station located in k that is sent to the incinerator located in l

y_k: binary variable that indicates whether there is a transfer station in municipality k, including the existing stations

z_l: binary variable that indicates whether the incinerator is located in municipality l

Parameters

c_u: annual unit transportation cost for uncompacted waste: 0.128571429 € tonne^{-1}km^{-1}

c_c: annual unit transportation cost for compacted waste: 0.045 € tonne^{-1}km^{-1}

$d_{j,k}$: distance matrix between municipalities j where the waste is generated and a transfer station located in municipality k

$d_{j,l}$: distance matrix between municipalities j where the waste is generated and the incinerator facility located in municipality l

$d'_{k,l}$: distance matrix between the transfer station k where the waste is collected and compacted and the incinerator facility located in municipality l

$f_{j,k}$: matrix with binary variables, indicating if a municipality j is located within a certain distance to a municipality k

$f_{j,l}$: matrix with binary variables, indicating if a municipality j is located within a certain distance to a municipality l

$g_{k,l}$: matrix with binary variables, indicating if a municipality j is located within a certain distance to a municipality l

m: penalty term for limiting the number of total transfer stations: 10^6

q_j: annual quantity of waste collected at municipality j

s_k: annual capacity of the transfer station: 182500 tonne

$w^0_{j,k}$: Existing transfer station k^1 - municipality j^1 connection

B Modified model

In this section, the proposed modified model is presented. Almost all assumptions used by Antunes et al. [1] hold; however, the following changes are implemented :

1. The m term is a virtual cost and it is removed from the objective function; thus only transportation costs are minimized.

2. A constraint is added into the model for limiting the number of transfer stations N_{TS} in the network. The minimum number is 6, as this is the number of existent transfer stations in the system.

3. The maximum distance between municipalities and transfer stations is increased from 25 km to 30 km.

Objective function

$$min\ C = \sum_{j\in J}\sum_{k\in K} c_u d_{j,k} q_j w_{j,k} + \sum_{j\in J}\sum_{l\in L} c_u d_{j,l} q_j v_{j,l} + \sum_{k\in K}\sum_{l\in L} c_c d'_{k,l} x_{k,l}$$

Constraints

$$\sum_{k\in K} w_{jk} + \sum_{l\in L} v_{jl} = 1 \qquad \forall j \in J$$

$$\sum_{j\in J} q_j w_{jk} = \sum_{l\in L} x_{kl} \qquad \forall k \in K$$

$$w_{jk} \leq f_{jk} y_k \qquad \forall j \in J, k \in K$$

$$v_{jl} \leq f_{jl} z_l \qquad \forall j \in J, l \in L$$

$$x_{kl} \leq g_{kl} q z_l \quad \forall k \in K, l \in L$$

$$\sum_{j\in J} q_j w_{jk} \leq s_k y_k \quad \forall k \in K$$

$$\sum_{l\in L} z_l = 1$$

$$w_{jk} = w^0_{jk} \quad \forall j \in J^1, k \in K^1$$

$$\sum_{k\in K} y_k \leq N_{TS} \qquad N_{TS} \geq 6$$

$$w_{jk}, v_{jl}, y_k, z_l \in \{0,1\}, x_{kl} \geq 0 \qquad \forall j \in J, k \in K, l \in L$$

Decision variables

$v_{j,l}$: binary variable that indicates if the municipality j hauls uncompacted waste to the incinerator located in l

$w_{j,k}$: binary variable that indicates if the municipality j hauls uncompacted waste to a transfer station located in k

$x_{k,l}$: quantity of waste processed from the connected municipalities at the transfer station located in k that is sent to the incinerator located in l

y_k: binary variable that indicates whether there is a transfer station in municipality k, including the existing stations

z_l: binary variable that indicates whether the incinerator is located in municipality l

Parameters

c_u: annual unit transportation cost for uncompacted waste: 0.128571429 € tonne^{-1}km^{-1}

c_c: annual unit transportation cost for compacted waste: 0.045 € tonne^{-1}km^{-1}

$d_{j,k}$: distance between municipalities j where the waste is generated and a transfer station located in municipality k

$d_{j,l}$: distance between municipalities j where the waste is generated and the incinerator facility located in municipality l

$d'_{k,l}$: distance between the transfer station k where the waste is collected and compacted and the incinerator facility located in municipality l

$f_{j,k}$: matrix with binary variables, indicating if a municipality j is located within a certain distance to a municipality k

$f_{j,l}$: matrix with binary variables, indicating if a municipality j is located within a certain distance to a municipality l

$g_{k,l}$: matrix with binary variables, indicating if a municipality j is located within a certain distance to a municipality l

N_{TS}: Number of transfer stations in the network

q_j: annual quantity of waste collected at municipality j

s_k: annual capacity of the transfer station: 182500 tonne

$w^0_{j,k}$: Existing transfer station k^1 - municipality j^1 connection

C Extended model

In this section, the extended model proposed for this project is detailed. This model considers the inclusion of recycling centres, and penalizes the proximity between facilities (transfer stations and recycling centres) using a weight parameter. The waste generation data used for this extension is from 2019 [11, 12, 13, 16]

Objective function

$$min\ C = \sum_{j\in J}\sum_{k\in K} c_u d_{j,k} q_j w_{j,k} + \sum_{j\in J}\sum_{l\in L} c_u d_{j,l} q_j v_{j,l} + \sum_{k\in K}\sum_{l\in L} c_c d'_{k,l} x_{k,l} + \sum_{j\in J}\sum_{r\in R} c_u d_{j,r} q_j^{rec} u_{j,r} - \sum_{j\in J}\sum_{k\in K} W d_{j,k} a_{j,k} - \sum_{j\in J}\sum_{r\in R} W d_{j,r} b_{j,r} \tag{1}$$

Constraints

$$\sum_{k\in K} w_{jk} + \sum_{l\in L} v_{jl} = 1 \qquad \forall j \in J$$

$$\sum_{j\in J} q_j w_{jk} = \sum_{l\in L} x_{kl} \qquad \forall k \in K$$

$$w_{jk} \leq f_{jk} y_k \qquad \forall j \in J, k \in K$$

$$v_{jl} \leq f_{jl} z_l \qquad \forall j \in J, l \in L$$

$$u_{jr} \leq f'_{jr} y_r^{rec} \qquad \forall j \in J, r \in R$$

$$x_{kl} \leq g_{kl} q z_l \quad \forall k \in K, l \in L$$

$$\sum_{j\in J} q_j w_{jk} \leq s_k y_k \quad \forall k \in K$$

$$\sum_{j\in J} q_j^{rec} u_{jr} \leq s_r y_r^{rec} \quad \forall r \in R$$

$$\sum_{l\in L} z_l = 1$$

$$w_{jk} = w_{jk}^0 \quad \forall j \in J^1, k \in K^1$$

$$a_{j,k} \leq y_j \quad \forall j \in J, k \in K$$

$$a_{j,k} \leq y_k \quad \forall j \in J, k \in K$$

$$b_{j,r} \leq y_j^{rec} \quad \forall j \in J, r \in R$$

$$b_{j,r} \leq y_r^{rec} \quad \forall j \in J, r \in R$$

$$\sum_{k \in K} y_k \leq N_{TS}$$

$$\sum_{r \in R} y_r^{rec} \leq N_{rec}$$

$$0 \leq a_{j,k} \leq 1 \quad \forall j \in J, k \in K$$

$$0 \leq b_{j,r} \leq 1 \quad \forall j \in J, r \in R$$

$$w_{jk}, v_{jl}, u_{jr}, y_k, y_r^{rec}, z_l \in \{0,1\}, x_{kl} \geq 0 \qquad \forall j \in J, k \in K, l \in L, r \in R$$

Decision variables

$w_{j,k}$: binary variable that indicates if the municipality j hauls uncompacted waste to a transfer station located in k

$v_{j,l}$: binary variable that indicates if the municipality j hauls uncompacted waste to the incinerator located in l

$u_{j,r}$: binary variable that indicates if the municipality j hauls recyclable waste to a recycling center located in r

$x_{k,l}$: quantity of waste processed from the connected municipalities at the transfer station located in k that is sent to the incinerator located in l

y_k: binary variable that indicates whether there is a transfer station in municipality k, including the existing stations

z_l: binary variable that indicates whether the incinerator is located in municipality l

y_r^{rec}: binary variable that indicates whether there is a recycling centre in municipality r

$a_j k$: variable indicates whether there is a transfer station in municipality j and another one in k

$b_j r$: variable indicates whether there is a recycling centre in municipality j and another one in r

Parameters

c_u: annual unit transportation cost for uncompacted waste: 0.173532 € tonne^{-1}km^{-1} (updated value to the 2019 using cost indices [10])

c_c: annual unit transportation cost for compacted waste: 0.060736 € tonne^{-1}km^{-1} (updated value to the 2019 using cost indices [10])

$d_{j,k}$: distance between municipalities j where the waste is generated and a transfer station located in municipality k

$d_{j,l}$: distance between municipalities j where the waste is generated and the incinerator facility located in municipality l

$d'_{k,l}$: distance between the transfer station k where the waste is collected and compacted and the incinerator facility located in municipality l

$d_{j,r}$: distance between municipalities j where the waste is generated and a recycling centre located in municipality r

$f_{j,k}$: matrix with binary variables, indicating if a municipality j is located within a certain distance to a municipality k

$f_{j,l}$: matrix with binary variables, indicating if a municipality j is located within a certain distance to a municipality l

$f'_{j,r}$: matrix with binary variables, indicating if a municipality j is located within a certain distance to a municipality r

$g_{k,l}$: matrix with binary variables, indicating if a municipality j is located within a certain distance to a municipality l

q_j: annual quantity of non-recyclable waste collected at municipality j

q_j^{rec}: annual quantity of recyclable waste collected at municipality j

s_k: annual capacity of the transfer station: 182500 tonne

s_r: annual capacity of the recycling centre: 182500 tonne

N_{TS}: total number of transfer stations: 9

N_{rec}: total number of recycling centres: 9

W: weight parameter for the dispersion of the facilities: 100

C.1 Sensitivity analysis

A sensitivity analysis was done on the maximum distance for hauling uncompacted waste. The following table details the results obtained:

Table 4: Results

Maximum distance (km)	Incinerator	Transfer stations	Recycling centres	Transportation cost (€x10^3/year)
25	Mealhada	Ansiao Coimbra* Estarreja Gois Ilhavo* Montemor-o-Velho* Oliveira de Azemeis Pampilhosa da Serra Sever do Vouga	Aveiro Cantanhede Condeixa-a-Nova Figueira da Foz Figueiro dos Vinhos Gois Ovar Pampilhosa da Serra Vale de Cambra	1757.9
30	Mealhada	Ansiao Aveiro* Coimbra* Estarreja Figueira da Foz* Gois Oliveira de Azemeis Pampilhosa da Serra Sever do Vouga	Arganil Arouca Aveiro Coimbra Figueira da Foz Figueiro dos Vinhos Ovar Oliveira de Azemeis Pampilhosa da Serra	1693.5
35	Mealhada	Aveiro* Ansiao Coimbra* Estarreja Figueira da Foz* Gois Oliveira de Azemeis Pampilhosa da Serra Sever do Vouga	Aveiro Cantanhede Condeixa-a-Nova Figueira da Foz Figueiro dos Vinhos Gois Ovar Pampilhosa da Serra Vale de Cambra	1695.9

* New transfer stations

C.2 Only one facility in a municipality

After reviewing the results from the extended model, it can be seen that a municipality may have recycling centres and transfer stations. This result could cause opposition from the population in the municipality since having more than one obnoxious facility may not be accepted.

An additional constraint is considered that restricts the number of a facilities in the same municipality.

$$x_k + y_k + z_k \leq 1 \quad \forall\, k \in K$$

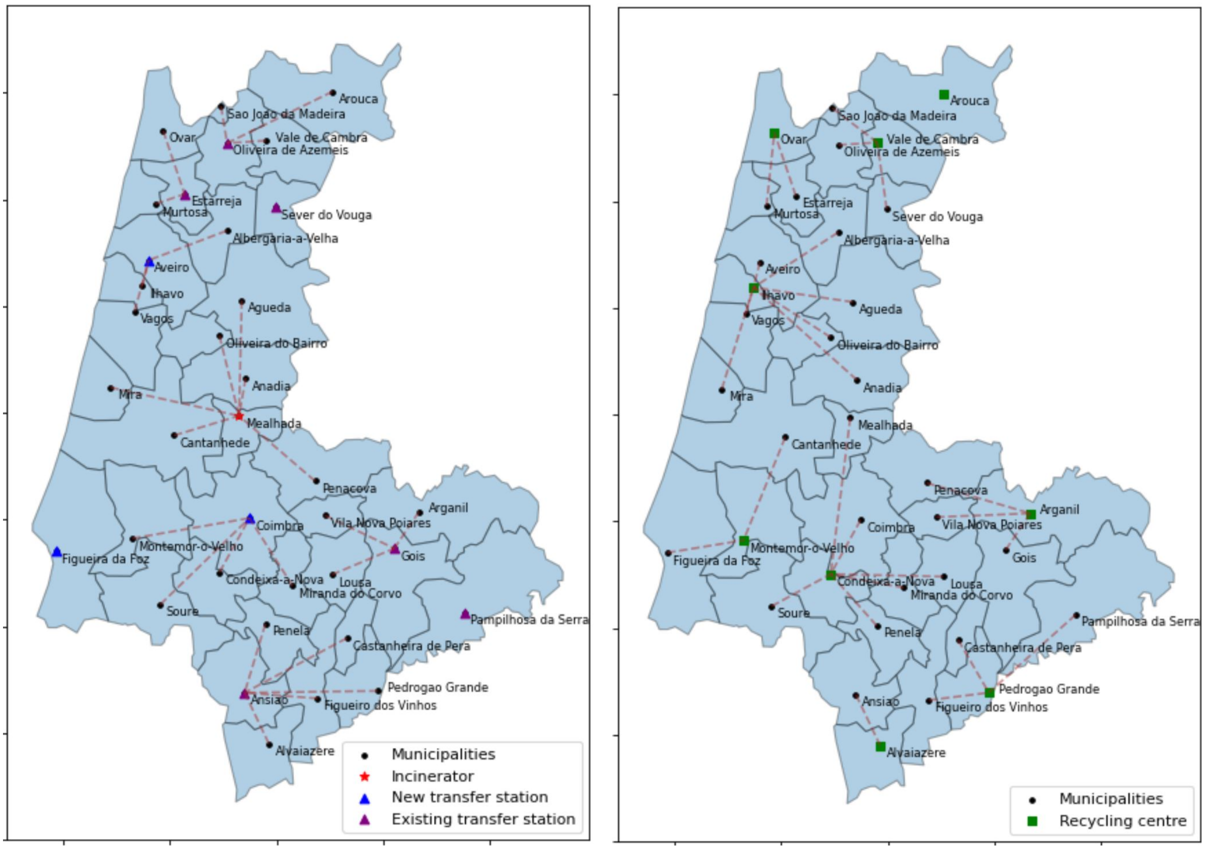
35 km is used as the maximum distance instead since 30 km leads to an infeasible solution. The results are obtained in approximately 2 minutes. The network configuration is detailed in the table below:

Table 5: Results

Maximum distance (km)	**Incinerator**	**Transfer stations**	**Recycling centres**	**Transportation cost (€x10^3/year)**
35	Mealhada	Ansiao Aveiro* Coimbra* Estarreja Figueira da Foz* Gois Oliveira de Azemeis Pampilhosa da Serra Sever do Vouga	Alvaizere Arganil Arouca Condeixa-a-Nova Ilhavo Montemor-o-Velho Ovar Pedrogao Grande Vale de Cambra	1738.9

* New transfer stations

The network visualization non-recycling and recycling facilities can be found in the below figure:



(a) Transfer station and incinerator network

(b) Recycling centres network

Figure 3: Network visualization using Geopandas [14]

D Program

A software solution was developed for running the models studied in this project. The solution front-end is a `Jupyter notebook` [15] which calls a helper `Python` script (`get_basura.py`) and interfaces to FICO Xpress 8.7 (Xpress Optimizer version 34.01.05) in the background [5] using Pyomo [8]. The program uses the following open source libraries:

1. Geopandas [14]: Geospatial support for data analysis library
2. Matplotlib [9]: 2D plotting library
3. Numpy [7]: Array handling library
4. Pandas [18]: Data analysis and manipulation library

5. Shapely [6]: Manipulation and analysis of geometric objects

The solution and its required dependencies can be found in the following repository: https://github.com/chiclez/MSW_Portugal [2]. The solution workflow is described in the flowchart below:

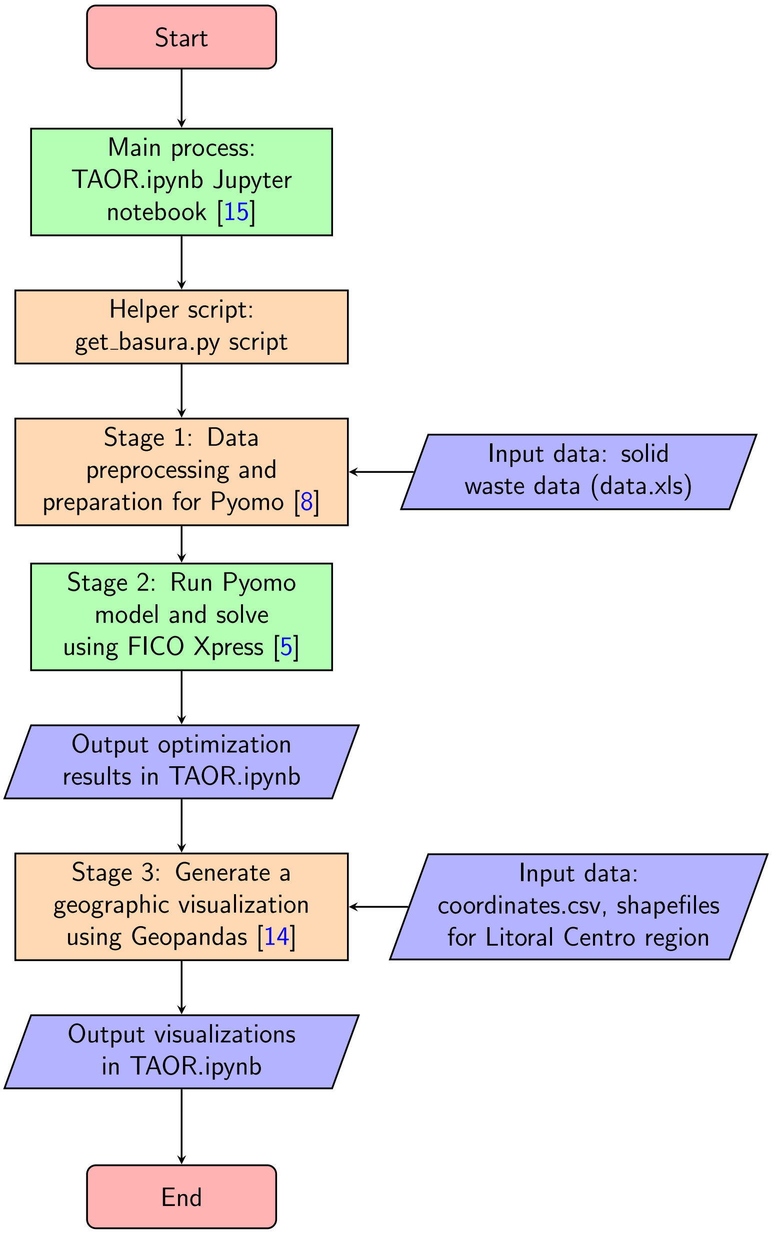


Figure 4: Program flowchart